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OPERATION APPROACHES ON SPECIFIC OPEN SETS

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Abstract: In this paper, we shall define the properties for arbitrary topological spaces such as $\alpha_{(\lambda,\lambda')}$-connected spaces and μ_Ω-compact spaces and shall prove the appropriate theorems with counter examples.Also we established that,$\alpha_{(\lambda,\lambda')}$-locally connectedness is an $\alpha_{(\lambda,\lambda')}$-open hereditary and we conclude that γ-operation defined by Ogata is an restriction of λ-operation.

Keywords and Phrases: $\alpha_{(\lambda,\lambda')}$-open sets, $\alpha_{(\lambda,\lambda')}$-connected spaces, Ω-open sets, μ_Ω-open sets and μ_Ω-compact spaces.

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1. Introduction

In 1979, Kasahara [15] initiated the concept of an operator (α) associated to a topology and gave some definitions which are equivalent to the usual ones when the operator involved is the identity operator. In 1991, Ogata [24] called the operation α as γ-operation on τ and defined and investigated the concept of operation-open sets, that is γ-open sets. He defined the complement of a γ-open subset of a space X as γ-closed. He also proved that the union of any collection of γ-open sets is γ-open but the intersection of any two γ-open sets need not be γ-open. Therefore the collection of γ-open sets need not be a topology on X. Krishnan. et al. [17] and [18] defined two types of sets called γ-preopen and γ-semiopen sets of a topological space (X, τ) respectively. Kalaivani. et al. [14] defined the notion of α-γ-open sets. Basu. et al. [8] used the operation γ to introduce γ-β-open sets. Also Carpintero. et al. [10] defined the notation of γ-b-open sets of a topological space (X, τ). In 2013, Barvan A. Asaad. et al. [3] initiated a new class of sets called γ-regular open sets in a topological space (X, τ) with an operation γ on τ together with its complement which is γ-regular closed. Also they defined a new space called γ-extremally disconnected and obtained several characterizations for γ-extremally disconnected spaces by utilizing γ-regular open sets and γ-regular closed sets. In 1965, Njastad [23] defined and investigated a new class of generalized open sets in a topological space called α-open sets. H. Z. Ibrahim [13] established and discussed an operation of a topology $\alpha O(X)$ into the power set $\mathcal{P}(X)$ of a space X and also he introduced the concept of α_γ-open sets. In 1992, Umehera. et al. [27] defined and discussed the properties of (γ, γ')-open sets. Khalaf. et al. [16] initiated the notion of $\alpha O(X, \tau)_{(\gamma,\gamma')}$, which is the collection of all $\alpha_{(\gamma,\gamma')}$-open sets in a topological space (X, τ). In 2018, Hariwan Zikri Ibrahim [12] defined and investigated the idea of $\alpha O(X, \tau)_{(\gamma,\gamma')}$ which is the assortment of all $\alpha_{(\gamma,\gamma')}$-open sets by using the operations γ and γ' on a topological space $\alpha O(X, \tau)$. Levine [26] initiated the concept of semi-open sets in topology. In 1987, Bhattacharyya. et al. [9] used semi-open sets to define the notion of semi-generalized closed sets. Assad [2] initiated and discussed the notion of an operation γ on the class of generalized open sets in (X, τ) and studied some of its applications. In 2017 - 2018, Ahmad and Assad ([1], [5]) introduced an operation γ on the class of semi generalized open sets of X and discussed some types of separation axioms, functions and closed spaces with respect to γ. Asaad and Ameen [6] introduced an operation on the class of generalized α - open sets and studied some of its properties. Recently, Assad et al. introduced the notion of an operation γ on the class of supra open sets in topological spaces, and investigated some important properties of it. The concept Ω-closed set was instigated and discussed by Lellis Thivagar [19]. In 2019, Meenarani. et al. [21] defined the concept of an operation γ which was extended to the class of Ω-open sets and it leads to the initiation of the notion of γ_Ω-open sets on a topological spaces (X, τ) and derivation of some basic properties of γ_Ω-closure. The definition of connectedness for a space is natural one and the notion of connectedness is not nearly so natural as that of compactness. In this paper, we define $\alpha_{(\lambda,\lambda')}$-connected spaces and we study μ_Ω-compact spaces associated with a topology. Also we examine some characterizations of $\alpha_{(\lambda,\lambda')}$-connected spaces and give some properties of μ_Ω-compact spaces using stable operator.

2. Preliminaries

Definition 2.1. *Let (X, τ) be a topological space and let M be a subset of X. Then M is $\alpha-$open if $M \subset int(cl(int(M)))$. The collection of all $\alpha-$open sets is denoted by $\alpha O(X)$.*

Definition 2.2. *A mapping $\lambda : \alpha O(X) \mapsto \mathcal{P}(X)$ is an operation if $M \subset \lambda(M)$ for all $M \in \alpha O(X)$, where $\lambda(M)$ is the value of λ at M.*

Definition 2.3. *Let (X, τ) be a topological space and let $M \subset X$. Then M is*

(1) (λ, λ')-open [21] ($\alpha_{(\lambda,\lambda')}$-open [7]) if for all $x \in M$, there exist open sets (respectively α-open sets) P and Q containing x such that $\lambda(P) \cup \lambda'(Q) \subset M$.

(2) $[\lambda, \lambda']$ - open [21] ($\alpha_{[\lambda,\lambda']}$-open [21]) if for all $x \in M$, there exist open sets (respectively α - open sets) P and Q containing x such that $\lambda(P) \cap \lambda'(Q) \subset M$.

The following example shows that open and $\alpha_{(\lambda,\lambda')}$-open are independent.

Example 2.4. Let $X = \{1, 2, 3, 4\}$ and let $\tau = \{\emptyset, X, \{4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ be a topology on X. Define the operations λ and λ' by
$\lambda(M) = \begin{bmatrix} M & \text{if } M \in \{\{1,4\}, \{1,3,4\}\} \\ cl(M) & \text{otherwise} \end{bmatrix}$ and $\lambda'(M) = \begin{bmatrix} cl(M) & \text{if } M = \{2,4\} \\ M & \text{if } M \neq \{2,4\} \end{bmatrix}$.
Here $\{2, 4\}$ is open but not $\alpha_{(\lambda,\lambda')}$-open and $\{1, 4\}$ is $\alpha_{(\lambda,\lambda')}$-open but not open.

3. $\alpha_{(\lambda,\lambda')}$-Connected Spaces

Using $\alpha_{(\lambda,\lambda')}$-open sets, we define $\alpha_{(\lambda,\lambda')}$-connected spaces and examine some of its characterizations.

Definition 3.1. *A topological space (X, τ) with operations λ and λ' on $\alpha O(X)$ is $\alpha_{(\lambda,\lambda')}$-**connected** if there is no disjoint pair M and N of $\alpha_{(\lambda,\lambda')}$-open sets such that $M \cup N = X$.*

Example 3.2.

(1) In (R, τ) where R is the set of real numbers and τ is the usual topology on R, the α-open sets are of the form (a, b). Define $\lambda = \lambda' : \alpha O(X) \to \mathcal{P}(X)$ by $\lambda(M) = \lambda'(M)$ =M for all $M \in \alpha O(X)$. In this topological space the $\alpha_{(\lambda,\lambda')}$-open sets are of the form (a, b). Therefore there does not exists a pair of disjoint $\alpha_{(\lambda,\lambda')}$-open sets M and N such that $M \cup N = X$. Thus (R, τ) is $\alpha_{(\lambda,\lambda')}$-connected.

(2) R with lower limit topology is not $\alpha_{(\lambda,\lambda')}$-connected under the same operations λ and λ' defined in the above example because the $\alpha_{(\lambda,\lambda')}$-open sets are of the form (a, b) and $[a, b)$. Therefore $(-\infty, 1) \cap [1, \infty) = \emptyset$ and $(-\infty, 1) \cup [1, \infty) = R$.

Definition 3.3. *Let (X, τ) be a topological space and $Y \subset X$. Then the collection of $\alpha_{(\lambda,\lambda')}$-open sets in Y is $\tau_{\alpha_{(\lambda,\lambda')Y}} = \{Y \cap M : M$ is $\alpha_{(\lambda,\lambda')}$-open in $X\}$.*

Remark 3.4.

(a) *A topological space (X,τ) is $\alpha_{(\lambda,\lambda')}$-disconnected if and only if there exists a non-empty subset M of X which is both $\alpha_{(\lambda,\lambda')}$-open and $\alpha_{(\lambda,\lambda')}$-closed.*

(b) *Union of $\alpha_{(\lambda,\lambda')}$-connected subsets of X having a point in common is $\alpha_{(\lambda,\lambda')}$-connected.*

(c) *Let P be a $\alpha_{(\lambda,\lambda')}$-connected subset of X and $M \subset X$ such that $P \subset M \subset \alpha_{(\lambda,\lambda')}$-cl(P). Then M is $\alpha_{(\lambda,\lambda')}$-connected.*

Definition 3.5. *For a subset M of a topological space X, the* $\alpha_{(\lambda,\lambda')}$**-boundary** *of M is equal to $\alpha_{(\lambda,\lambda')}$-cl(M) $\cap$ $\alpha_{(\lambda,\lambda')}$-cl($M^c$).*

In example 3.2, let $M = (-2, 0)$. Then the boundary of M is $\{-2, 0\}$.

The following theorem gives a characterization for $\alpha_{(\lambda,\lambda')}$-connectedness in terms of $\alpha_{(\lambda,\lambda')}$-boundary.

Theorem 3.6. *A topological space (X,τ) is $\alpha_{(\lambda,\lambda')}$-connected if and only if every non-empty proper subset of X has a non-empty $\alpha_{(\lambda,\lambda')}$-boundary.*

Proof. Assume that (X,τ) is $\alpha_{(\lambda,\lambda')}$-connected. Let M be a proper non-empty proper subset of X such that M has empty $\alpha_{(\lambda,\lambda')}$-boundary. Then $\alpha_{(\lambda,\lambda')}\text{-cl}(M) \cap \alpha_{(\lambda,\lambda')}\text{-cl}(X\backslash M) = \emptyset$.

$$\begin{aligned}
&\Rightarrow \quad \alpha_{(\lambda,\lambda')}\text{-cl}(M) \subset X\backslash \alpha_{(\lambda,\lambda')}\text{-cl}(X\backslash M)\\
&\Rightarrow \quad M \subset \alpha_{(\lambda,\lambda')}\text{-cl}(M) \subset \alpha_{(\lambda,\lambda')}\text{-int}(M) \subset M.\\
&\Rightarrow \quad M = \alpha_{(\lambda,\lambda')}\text{-cl}(M) = \alpha_{(\lambda,\lambda')}\text{-int}(M).
\end{aligned}$$

Therefore M is both $\alpha_{(\lambda,\lambda')}$-open and $\alpha_{(\lambda,\lambda')}$-closed. That is X is not $\alpha_{(\lambda,\lambda')}$-connected which is a contradiction.
Conversely, Suppose X is $\alpha_{(\lambda,\lambda')}$-disconnected. Then X has a proper subset M which is both $\alpha_{(\lambda,\lambda')}$-open and $\alpha_{(\lambda,\lambda')}$-closed.

$$\begin{aligned}
&\Rightarrow \quad \alpha_{(\lambda,\lambda')}\text{-cl}(M) = M \text{ and } \alpha_{(\lambda,\lambda')}\text{-cl}(X\backslash M) = X\backslash M\\
&\Rightarrow \quad \alpha_{(\lambda,\lambda')}\text{-cl}(M) \cap \alpha_{(\lambda,\lambda')}\text{-cl}(X\backslash M) = \emptyset.
\end{aligned}$$

which is a contradiction. Therefore X is $\alpha_{(\lambda,\lambda')}$-connected.

Definition 3.7. *$\alpha_{(\lambda,\lambda')}$-component of X is a* **maximal** $\alpha_{(\lambda,\lambda')}$**-connected** *subset of X.*

Example 3.8. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\},$

$\{b, c\}$, $\{a, b, c\}\}$. Define λ and λ' on $\alpha O(X)$ by

$$\lambda(M) = \begin{cases} \text{cl(M)} & \text{if } d \in M \\ M & \text{if } d \notin M \end{cases}$$

and

$$\lambda'(M) = \begin{cases} M & \text{if } d \in M \\ \text{cl (M)} & \text{if } d \notin M \end{cases}$$

In this topological space {a,b,c} is not $\alpha_{(\lambda,\lambda')}$-component while {a,c,d} is $\alpha_{(\lambda,\lambda')}$-component of X.

Theorem 3.9. *For a topological space (X, τ), the following conditions hold*

(a) There is exactly one $\alpha_{(\lambda,\lambda')}$-component of X containing x for each $x \in X$.

(b) Each $\alpha_{(\lambda,\lambda')}$-connected subset of X is contained in exactly one $\alpha_{(\lambda,\lambda')}$-component of X.

(c) If the operations λ and λ' are α-regular, then every $\alpha_{(\lambda,\lambda')}$-connected subset of X which is both $\alpha_{(\lambda,\lambda')}$-open and $\alpha_{(\lambda,\lambda')}$-closed is $\alpha_{(\lambda,\lambda')}$-component of X.

(d) Every $\alpha_{(\lambda,\lambda')}$-component of X is $\alpha_{(\lambda,\lambda')}$-closed subset of X.

Proof.

(a) Let $x \in X$ and $\mathcal{P} = \{P_i : i \in I\}$ be a collection of $\alpha_{(\lambda,\lambda')}$-connected subsets of X containing x. Take $P = \cup P_i$. Then P is $\alpha_{(\lambda,\lambda')}$-connected and $x \in P$. Suppose $P \subset P^*$ for some $\alpha_{(\lambda,\lambda')}$-connected subset P^* of X. Then $x \in P^*$ and P^* is an element of $\mathcal{P}$. This implies $P^* \subset P$. That is $P^* = P$. Therefore P is $\alpha_{(\lambda,\lambda')}$-component of X.

(b) Let P be a $\alpha_{(\lambda,\lambda')}$-connected subset of X which is not a $\alpha_{(\lambda,\lambda')}$-component of X. Suppose P_1 and P_2 are $\alpha_{(\lambda,\lambda')}$-components of X such that $P \subset P_1$ and $P \subset P_2$. Since $P \subset P_1 \cap P_2$, that is $P_1 \cap P_2 \neq \emptyset$. Therefore $P_1 \cup P_2$ is $\alpha_{(\lambda,\lambda')}$-connected and contains P_1 and P_2. This is a contradiction to P_1 and P_2 are $\alpha_{(\lambda,\lambda')}$-components. Therefore P is contained in exactly one $\alpha_{(\lambda,\lambda')}$-component of X.

(c) Suppose M is $\alpha_{(\lambda,\lambda')}$-connected subset of X which is both $\alpha_{(\lambda,\lambda')}$-open and $\alpha_{(\lambda,\lambda')}$-closed. By (b), M is contained in exactly one $\alpha_{(\lambda,\lambda')}$-component P of X. If M is a proper subset of P, then $P = (P \cap M) \cup (P \cap (X \backslash M))$ is a $\alpha_{(\lambda,\lambda')}$-disconnection of P. This is a contradiction. Thus $M = P$. Therefore M is $\alpha_{(\lambda,\lambda')}$-component of X.

(d) Suppose the $\alpha_{(\lambda,\lambda')}$-component P of X is not $\alpha_{(\lambda,\lambda')}$-closed. Then $\alpha_{(\lambda,\lambda')}$-cl(P) is $\alpha_{(\lambda,\lambda')}$-connected and containing $\alpha_{(\lambda,\lambda')}$-component P of X. This implies $P = \alpha_{(\lambda,\lambda')}$-cl(P). Therefore P is $\alpha_{(\lambda,\lambda')}$-closed.

Definition 3.10. *A topological space* (X, τ) *is* $\alpha_{(\lambda,\lambda')}$**-locally connected** *if for every point* $x \in X$ *and every* $\alpha_{(\lambda,\lambda')}$*-open set* M *containing* x*, there is a* $\alpha_{(\lambda,\lambda')}$*-connected and* $\alpha_{(\lambda,\lambda')}$*-open set* O *such that* $x \in O \subset M$*.*

Example 3.11. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$. Define λ and λ' on $\alpha O(X)$ by

$$\lambda(M) = \begin{cases} M & \text{if } d \in M \\ M \cup \{d\} & \text{if } d \notin M \end{cases}$$

and

$$\lambda'(M) = \begin{cases} M & \text{if } c \in M \\ M \cup \{c\} & \text{if } c \notin M \end{cases}$$

In this topological space, $\alpha_{(\lambda,\lambda')}$-open sets are $\emptyset$, X and {a,c,d}. Clearly X is $\alpha_{(\lambda,\lambda')}$-locally connected.

Theorem 3.12. *If* X *is* $\alpha_{(\lambda,\lambda')}$*-locally connected, then* X *has a base with* $\alpha_{(\lambda,\lambda')}$*-connected and* $\alpha_{(\lambda,\lambda')}$*-open sets.*

Proof. Let $\mathcal{M}$ be the collection of all $\alpha_{(\lambda,\lambda')}$-connected and $\alpha_{(\lambda,\lambda')}$-open subsets of X. Let $x \in X$ and let O be $\alpha_{(\lambda,\lambda')}$-open set containing x. Since X is $\alpha_{(\lambda,\lambda')}$-locally connected, there exists $\alpha_{(\lambda,\lambda')}$-connected and $\alpha_{(\lambda,\lambda')}$-open set $M \in \mathcal{M}$ such that $x \in M \subset O$. This implies O is the union of elements of $\mathcal{M}$. Therefore $\mathcal{M}$ is a base for the collection of all $\alpha_{(\lambda,\lambda')}$-open sets of X.

Theorem 3.13. $\alpha_{(\lambda,\lambda')}$*-locally connectedness is* $\alpha_{(\lambda,\lambda')}$*-open hereditary.*

Proof. Let M be $\alpha_{(\lambda,\lambda')}$-open subset of X containing x and let O be $\alpha_{(\lambda,\lambda')}$-open set such that $x \in O \subset M$. Then O is $\alpha_{(\lambda,\lambda')}$-open set containing x in X. Since X is $\alpha_{(\lambda,\lambda')}$-locally connected, there exists $\alpha_{(\lambda,\lambda')}$-open set W containing x such that $x \in W \subset O$. W is $\alpha_{(\lambda,\lambda')}$-connected and $\alpha_{(\lambda,\lambda')}$-open set containing x such that $x \in W \subset O \subset M$. Therefore M is $\alpha_{(\lambda,\lambda')}$-locally connected.

Example 3.14.

(1) Sierpinski space, $(X = \{0, 1\}, \tau = \{\phi, X, \{1\}\})$, with the operations defined by $\lambda(M)$ =cl (M) and $\lambda'(M)$ =M is both $\alpha_{(\lambda,\lambda')}$-connected and $\alpha_{(\lambda,\lambda')}$-locally connected.

(2) R, the set of real numbers with discrete topology on R and the operations defined on $\alpha O(R)$ by $\lambda(M)$=cl(M) and $\lambda'(M)$=M is neither $\alpha_{(\lambda,\lambda')}$-connected nor $\alpha_{(\lambda,\lambda')}$-locally connected.

(3) In R with usual topology with operations $\lambda(M)$=M and $\lambda'(M)$=cl (M), the subspace $Y = [-1, 0) \cup (0, 1] \subset R$ is $\alpha_{(\lambda,\lambda')}$-locally connected but not $\alpha_{(\lambda,\lambda')}$-connected.

4. μ_Ω-Compact Spaces

μ_Ω-compact space was defined and some properties of μ_Ω-compact spaces were examined in this section.

Lemma 4.1. *γ-operation is a restriction of λ-operation.*
Proof. Let M be an open set. Then M is an α-open set. Therefore $\tau \subset \alpha O(X)$. Thus, $\lambda_\tau = \gamma$.

It is clear that λ-operation need not be γ-operation.

Example 4.2. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. In this topological space α-open sets are $\emptyset$, X, {a}, {d}, {a,d}, {b,d}, {a,b,d}, {a,c,d}.

In the above example Ω-open sets are {b,c,d} and {b,d}.

Here {b,c,d} is Ω-open but not open and {d} is open but not Ω-open.

Therefore open and Ω-open are independent.

Similarly {a,c,d} is α-open but not Ω-open and {b,c,d} is Ω-open but not α-open. Therefore α-open and Ω-open are independent.

Theorem 4.3. *Let (X, τ) be a topological space with an operation γ on τ, $Y \subset X$ and let M be a γ-open subset of X. Then $M \cap Y$ is γ-open in Y if $\gamma(O) \subset M \Rightarrow \gamma(O \cap Y) \subset M \cap Y$ for all $O \in \tau$.*
Proof. Let M be a γ-open subset in X. Then for all $x \in M$ there exists an open set O of X containing x such that $\gamma(O) \subset M$. By assumption, $\gamma(O \cap Y) \subset M \cap Y$. Since $O \cap Y$ is open in Y and $M \cap Y \subset M$, for all $x \in M \cap Y$ there exists an open set O of X such that $\gamma(O \cap Y) \subset M \cap Y$. Thus $M \cap Y$ is γ-open in Y.

Corollary 4.4. *If Y is open in X, then γ-open sets in Y are γ-open in X.*

Definition 4.5. *An operation μ on $\Omega O(X)$ is* **stable** *if μ induces an operation $\mu_Y : \Omega_Y O(X) \to \mathcal{P}(X)$ such that $\mu_Y(M \cap Y) = \mu(M) \cap Y$ for every $M \in \Omega O(X)$ where $\Omega_Y O(X) = \{M \cap Y : M \in \Omega O(X)\}$.*

Example 4.6. Let $X = \{a, b, c, d\}$ and let $\tau = \{\emptyset, X, \{a\}, \{c\}\{a, b\}, \{a, c\}, \{a, b, c\}\}$. In this topological space, Ω-open sets are {c},{a,b},{a,c},{a,b,c}. Define μ on $\Omega O(X)$ by

$$\mu(M) = \begin{cases} M \cup \{b\} & \text{if } M = \{d\} \\ M & \text{if } M \neq \{d\} \end{cases}$$

Let Y={b,c,d}. Then the operation μ is stable with respect to Y.

Proposition 4.7. *Let $\mu : \Omega O(X) \to \mathcal{P}(X)$ be an operation satisfying the following conditions:*

(1) $M \subset \mu(M)$ for all $M \in \Omega O(X)$.

(2) $\mu(\emptyset) = \emptyset$.

If μ is stable with respect to all proper Ω-closed subsets of X, then μ is an identity operation.

Proof. Let M be a proper Ω-open subset of X. Then $M^c = P$ is Ω-closed. Since μ is stable,

$$\begin{aligned} \mu(M) \cap P &= \mu_P(M \cap P) \\ &= \mu_P(\emptyset) \\ &= \mu_P(\emptyset \cap P) \\ &= \mu(\emptyset) \cap P \\ &= \emptyset \cap P = \emptyset. \end{aligned}$$

This implies $\mu(M) \subset M$. Therefore $\mu(M) = M$.

Theorem 4.8. *Let (X, τ) be a topological space with an operation μ on $\Omega O(X)$, $Y \subset X$ and let M be a μ_Ω-open subset of X. Then $M \cap Y$ is μ_Ω-open in Y if $\mu(O) \subset M \Rightarrow \mu(O \cap Y) \subset M \cap Y$ for all $O \in \Omega O(X)$.*

Proof. Let M be a μ_Ω-open subset in X. Then for all $x \in M$ there exists an Ω-open set O of X containing x such that $\mu(O) \subset M$. By assumption, $\mu(O \cap Y) \subset M \cap Y$. Since $O \cap Y$ is Ω-open in Y and $M \cap Y \subset M$, for all $x \in M \cap Y$ there exists an Ω-open set O of X such that $\mu(O \cap Y) \subset M \cap Y$. Thus, $M \cap Y$ is μ_Ω-open in Y.

Definition 4.9. *Let (X, τ) be a topological space with an operation μ on $\Omega O(X)$. Then X is μ_Ω-compact if for every Ω-open cover $\mathcal{O}$ of X, there exists a finite sub collection $\{O_1, O_2, ..., O_n\}$ of $\mathcal{O}$ such that $X = \cup_{i=1}^{n} \mu(O_i)$.*

Example 4.10. Let (N, τ) be a discrete topological space where N is the set of natural numbers. Define the operation μ on $\Omega O(X)$ by $\mu(M) = N$ for all $M \in \mathcal{N}$. This topological space is μ_Ω-compact.

Remark 4.11. *μ_Ω-closed subset of μ_Ω-compact space is μ_Ω-compact.*

Theorem 4.12. *μ_Ω-compact subset of a μ_Ω-T_2 space is μ_Ω-closed if the operation is μ_Ω-regular.*

Proof. Let M be a μ_Ω-compact subset of X. Claim M is μ_Ω-closed. Let $x \in M^c$. For all $y \in M$, there exist Ω-open sets P and Q such that $x \in P$, $y \in Q$ and

$\mu(P) \cap \mu(Q) = \emptyset$. Using these Ω-open sets, we can construct a Ω-open cover $\{O_y : y \in M\}$ of M. Since M is μ_Ω-compact, there exists a finite collection $\{O_1, O_2, ...O_n\}$ such that $M \subset \cup_{i=1}^{n} \mu(O_i)$. Let $O = \cap_{i=1}^{n} O_i$. Then O is Ω-open set containing x. Since $O_1, O_2, ...O_n$ are Ω-open sets containing x and μ is μ_Ω-regular, there exists Ω-open set W such that $W \subset \mu(W) \subset M^c$. This implies M^c is μ_Ω-open. Therefore M is μ_Ω-closed.

Theorem 4.13. *Let (X, τ) be a topological space with an operation μ on $\Omega O(X)$ which is stable with respect to Y where $Y \subset X$. If X is μ_Ω-compact and Y is μ_Ω-closed, then Y is μ_{Ω_Y}-compact.*

Proof. Let $\phi = \{O_i\}_{i \in I}$ be a Ω-open cover of Y by $\Omega_Y O(Y)$-open sets. Let $\phi^* \subset \Omega O(X)$ be the set of all Ω-open sets such that for all $M \in \phi^*$, $M \cap Y \in \phi$. Since $X \backslash Y$ is μ_Ω-open, take a Ω-open cover of $X \backslash Y$ say $\chi = \{N_x \in \Omega O(X) : \mu(N_x) \subset X \backslash Y, x \in X\}$. Then the collection $\phi^* \cup \chi$ is a Ω-open cover of X. Since X is μ_Ω-compact, there exist a finite subcollection $\{O_1, O_2, \ldots, O_m\} \subset \phi^*$ and $\{N_1, N_2, \ldots, N_n\} \subset \chi$ such that

$$X = \{\cup_{i=1}^{m} \mu(O_i)\} \cup \{\cup_{j=1}^{n} \mu(N_j)\}.$$

Then

$$Y = \{\cup_{i=1}^{m} \mu(O_i) \cap Y\} \cup \{\cup_{j=1}^{n} \mu(N_j) \cap Y\}.$$

Since $\mu(N_j) \cap Y = \emptyset$ for $j = 1, 2, \ldots, n$ and μ is stable with respect to Y,

$$Y = \{\cup_{i=1}^{m} \mu_Y(O_i \cap Y)\}.$$

Therefore Y is μ_{Ω_Y}-compact.

Theorem 4.14. *Let M be a subset of X and let $\mu : \Omega O(X) \to \mathcal{P}(X)$ and $\mu_M : \Omega_M O(M) \to \mathcal{P}(M)$ be operations satisfying $\mu_M(O \cap M) \subset \mu(O) \cap M$ for any Ω-open subset O of X such that $M \cap O \neq \emptyset$. If M is μ_{Ω_M}-compact, then M is μ_Ω-compact.*

Proof. Let $\mathcal{O}$ be Ω-open cover of M. Then $\{O \cap M : O \in \mathcal{O}\} \subset \Omega_M O(M)$ is a Ω_M-open cover of M. Therefore there exists a finite collection $\{O_1, O_2, ...O_n\}$ of $\mathcal{O}$ such that $M = \cup_{i=1}^{n} \mu_M(O_i \cap M) \subset \cup_{i=1}^{n} \mu(O_i) \cap M \subset \cup_{i=1}^{n} \mu(O_i)$. Therefore M is μ_Ω-compact.

5. Conclusion

In this article, we proved that, In a topological space X, there is exactly one $\alpha_{(\lambda,\lambda')}$ component of X containing a particular element x. Also we established that, $\alpha_{(\lambda,\lambda')}$-locally connectedness is an $\alpha_{(\lambda,\lambda')}$-open hereditary and we conclude that γ-operation defined by Ogata is an restriction of λ-operation. We proven that Sierpinski space with particular operation is both $\alpha_{(\lambda,\lambda')}$-connected and $\alpha_{(\lambda,\lambda')}$-locally connected. Also we defined and investigated the properties of μ_Ω-compact spaces with illustrative examples.

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